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ON DEFINING MEANING FAMILIES

1. In his *Philosophical Investigations* (Oxford 1958, p. 31–35) Wittgenstein remarked that in natural languages there are expressions which are undefinable. The word "game" was given as an example of undefinable expressions. The meaning of this and similar words is to be grasped only by considering different kinds of games and by hearing the word in different contexts. Wittgenstein says that this word has a sort of meaning consisting of many similar meanings constituting a family of meanings. The remarks of Wittgenstein suggest that the multitude of meanings is the cause of the impossibility of defining terms having families of meaning as their meaning.

It seems that not only in colloquial languages do exist expressions having families of meanings as their meaning. Languages used in scientific discourse exhibit expressions of this same kind. I am going now to show with the help of an example taken from biology that languages used in scientific discourse are not in a better position than colloquial languages as far as expressions having meaning families as meaning are considered. It seems to me that we encounter the same difficulties in defining the expression "animal" as in defining "game".

2. Let us take a classification of living beings (for instance the classifications in *Encyclopedia Americana*, 1963 vol. 1, p. 700; vol. 22 p. 155 and 158) dividing this set into two subsets: the subset of animals and the subset of plants. Let the subset of animals include among others the following classes of organisms: Flagellatae, Rhizopodae, Amoebosporidiae, Ciliatae, Anthozoae, Spongiae. The subsets of plants includes among others the fungi, especially Schyzomycetes, Eumycophyta and the algae especially Euglenophyta. This and similar classifications of living beings are the most common.

Now there arises the problem of defining the set of animals including the types just mentioned. Here we meet with unsurmountable difficulties if we try to accomplish a definition characterizing the set of animals by citing features[1] common to all animals and only to animals. For instance we cannot declare animals to be

(1) all and only these beings which are motile.

And this because of the fact that some corals and sponges do not move and nevertheless are members of the kingdom of animals. What is more, there are plants being

[1] The notion "feature" is conceived here in an Aristotelian way: we are concerned with non-constructed features, especially an alternative of conjunctions of features is not conceived as a feature.

able to move independently. So (1) is a definiens simultaneously too broad and too narrow. Similar difficulties arise when taking animals as these beings which

(2) have at least one sense organ.

Some animals e.g. corals have no sense organs and they exhibit no sensitivity. On the contrary some plants, e.g. the mimosa, behave as if they had sense organs. Touched by some stimulating eatable fly it reacts quickly and efficiently.

(3) The impossibility of photosynthesizing food

is also not a decisive difference between animals and plants. Indeed mushrooms (a class of Eumycophyta) are heterotropic and are ranked among plants. The fact that animals

(4) do not grow during the whole life

cannot serve either. To be sure the majority of plants are growing through the whole of their lives, but not all and therefore we cannot take (4) as a definition of animals and only of animals. The majority of animals have cells enclosed in thin membranes. Plants as a rule have cells with thick cellulose outer layers. Notwithstanding

(5) having cells with thin membranes

is not a distinctive feature as some microbes have such cells and are rated among the plants. The same applies to some primitive Eumycophytae. The chemical composition known up to now cannot be taken into account in defining animals. Such substances as hemoglobin and chitin are very often found in animals' bodies, but not all animals have them and there are plants which produce these substances. So

(6) hemoglobin and chitin

cannot be taken as the definiens of a definition of animals.

Neither can an adequate definition of animals be given by stating that animal is an organism having all the mentioned features or having a conjunction of some of them. Such a definition would exclude all plants from the kingdom of animals, but some organisms reckoned among animals should be treated as plants. If one would try to define the kingdom of animals by stating that a living organism is to be an animal having this or that of the mentioned properties (alternative of features) he would have to include many plants into the kingdom of animals.

The difficulties encountered in defining the kingdom of animals promted many attempts to change the situation. Most often they tried to change the existing divisions of living beings into the kingdoms of animals and plants. But these changes did not change anything at all from the point of view of defining animals. Some of the changed divisions were charged with artificiality. Other divisions did not remove the described difficulties. Still there remained organisms reckoned among animals which by definition were to be plants and vice versa. There were even propositions to abandon the division into animals and plants altogether by introducing a third kingdom of mezozoa. But this did not alleviate the trouble as there arose the problem how to distinguish animals from the mezozoa and how to delimit plants from the mezozoa, i.e. there was the problem of defining animals, plants and additionally the mezozoa.

3. Owing to this difficulties some biologists concluded that no adequate definitions of the kindgom of animals is possible. Perhaps to their minds the only way to know what beings are animals is to enumerate concrete species and genera of living being which they want to be animal species and genera. This consequence is very similar to that given by Wittgenstein. It seems correct if only traditional types of definitions are taken into account, i.e. definitions characterizing a set of entities (and thereby the respective term) by giving at least one feature which is common to all members of the set and only to them. Having in mind only such definitions Wittgenstein was quite right.

The conclusions we obtained thus far on the ground of an example taken from biology may be extended onto various other fields. There arise difficulties with defining in all cases were one has to delimit a class of entities which gradually, almost in an invisible way, turn into entities of another class. What is more, these complications arise if one is willing to delimit one class from another by taking into account many features so as to make the demarcation most natural. Then it often happens that from the point of view of one feature a given entity has to be classified into one class and from the point of view of another property it has to be classified into another class. In short, there arise troubles when defining sets of entities having many properties and the definition aims at delimiting classes in the most natural way by taking into consideration a great number of attributes of the entities defined.

As we very often try to define sets of entities in a natural way we have very often to do with difficulties described above. It appears that at least some of the difficulties met in defining science (set of sciences), tragedy (set of very sad plays), species (set of sets of similar organisms) game (set of some entertainments), nation (set of several sets of people) and so on, have their source in taking into consideration only traditional definitional schemes in making definitions of meaning families.

4. As Wittgenstein is right in stating that the traditionally known schemes of defining do not allow to determine meaning families adequately the question is to be put whether there are ways of defining meaning families at all. To answer this question let us see how the notion of animal has been generalized in the process of broadening our knowledge about living beings. Let us consider on what ground newly discovered species of living beings were reckoned among animals. Perhaps this historical scrutiny will reveal new schemes of defining.

At the wery beginning of man's conscious reflections upon animals only mammals, birds, fishes, reptils and a few other types of living beings were considered as animals. All this organisms had somes features sharply distinguishing them from the plants then known. The animals then considered were all motile, they were heterotrophic, and they were sensitive. It was noticed that animals were made out of a more elastic material than the plants (the cells have thin and elastic membranes). These features correspond to some of the above mentioned definitions of animals. With the progress of science new organisms were discovered and people had to deal with the problem where to ascribe these new living beings. It seems that the problem was always settled in the same way: if the new organism was more like the already known animals than the already known plants it was rated among the animals. Similarly it was ranked among plants if it was more like the already known plants. Owing to this procedure a new set of animals was created and this set might be defined in this sketchy way: an animal is either a living being having the long known features motility, heterotrophicity, sensitivity etc. or it is similar to beings having these properties. Thus the clue for defining an enlarged set of animals is the notion of similarity.

Two entities are but similar (i.e. they are not identical) if they have the same properties in different degrees. This concept of similarity was the foundation of Hempel's and Oppenheim's *Der Typusbegriff im Lichte der neuen Logik: Untersuchungen zur Konstitutionsforschung und Psychologie*, Leiden 1936. Here we should like to take into account another notion of similarity: two entities are similar if they have more common features than different ones. It seems that this concept of similarity plays an essential role in defining the set of animals.

The way we outlined the definition of animals calls attention to two essential steps in this process:

a. first the characteristic features of some initial set were fixed,

b. then all the entities having more than half the characteristic features of the initial set were reckoned into an enlarged set.

Let the initial set be called "type". The set defined may be called "development of the type". The features characterizing the type may be denominated "typical features". Now we are able to proceed to give a definitional scheme for such sets (and therefore for respective terms) as the set of animals. The definiendum of the scheme will take the form: "x is a member of the development of the type W". Its shortened form is "$x \in D_W$". The features of the type W will be designated by "F_1", "F_2", ..., "F_n". To speak about undetermined typical features we shall use the variables "F_i", "F_j", ..., "F_k" etc., the letters "i", "j", "k" etc. being variables ranging over the set of natural numbers. To emphasize the fact that an entity x has a definite number n' of typical features unknown to us the following way of writing will serve "$x \in F_{i_1} \cdot x \in F_{j_2} \cdot \ldots \cdot x \in F_{k_n}$" or more briefly "$x \in F_{i_1}, F_{j_2}, \ldots, F_{k_n}$". This formula does mean that x has exactly n features but we do not bother whether they are $F_1, F_2, \ldots, F_k$ or $F_3, F_4, \ldots, F_m$ or $F_2, F_4, F_5, \ldots, F_n$. Now here is a more precise formulation of a. and b.:

a. $$\prod u\,(u \in W \equiv u \in F_1, F_2, \ldots, F_m)$$

b. $$\sum i, j, k, n \left[(x \in F_{i_1}, F_{j_2}, \ldots, F_{k_n}) \cdot (i, j, k, n \leqslant m) \cdot (n > (1/2) \times m)\right]$$

The condition a. is to the effect that every member of the type W has the properties $F_1, F_2, \ldots, F_m$. The condition b. states that the member x of the development of the type W has more than half the features of the type W, i.e. the members of D_W are more similar to the type than dissimilar. The scheme for the definition of animal is thence ready:

(1) $$x \in D_W \equiv \prod u \{(u \in W \equiv u \in F_1, F_2, \ldots, F_m) \cdot \sum i, j, k, n \left[(x \in F_{i_1}, F_{j_2}, \ldots, F_{k_n}) \cdot (i, j, k, n \leqslant m) \cdot (n > (1/2) \times m)\right]\}$$

It is obvious that we assume the logical terms in (1) to have their normal meaning Moreover it must be noted that not all the variables in the definiens of (1) not appearing in the definiendum are bound. This is so because what we have here is not a concrete definition, but a scheme of definitions.

The scheme (1) may be generalized in different directions. The conviction that two entities are similar only if one of them exhibits more than half the features of the other was the foundation of (1) and it was expressed by "$n > (1/2)\times m$". In some cases it may be useful to define the notion of similarity in a somewhat different way, namely when the type has an even number of typical features it is more convenient to admit that two entities are similar if one of them has at least half the features of the second one. In this case the last condition of (1) would have to give way to "$n \geqslant (1/2)\times m$". There are still other possibilities of determining the notion of similarity. Thus it would be useful not to introduce any definite fraction into the definition but rather to establish an undetermined fraction for instance "r/p". This fraction has to meet only one requirement: it has to make a whole natural number, if multiplied by m, i.e. the number of typical features. So the more general scheme is the following:

(2) $$x \in D_W \equiv \prod u \Big\{(u \in W \equiv u \in F_1, F_2, \ldots, F_m)\cdot \sum i, j, k, n \Big[(x \in F_{i_1}, F_{j_2}, \ldots, F_{k_n})\cdot \cdot(i, j, k, n \leqslant m)\cdot(n \geqslant (r/p)\times m)\cdot((r/p)\times m \in Nat\ N)\Big]\Big\}$$

Still other schemes must be introduced when one wants to define subsets of a division and to make explicit differences between the subsets of the division. Let us first consider a division of a set into two subsets only. To obtain a division in which the subsets exclude one another 1) we have to start from two types which exclude one another. 2) The types must be subsets of the class the subaltern genera of which we are going to define. 3) All the entities, members of the class devided have at least one typical faeture. 4) The entities mentioned in the definiendum must be a member of the class divided. 5) The entities reckoned among one of the subsets have to be more similar to none another than entities of other types. All these postulates may be ilustrated on the example of defining animals. Till now we have not considered the animals as the subset of a division. Now such treatement of the set of animals is possible and it is the more natural one. The divided set is that of living beings. The types are the set of animals long known and the set of plants long known. It is easily seen that all the conditions are fulfilled. To obtain the new definitional scheme we have to introduce a new definiendum: x is a member of the development of the type W_1 which is one of the subsets of a twofold division of the set W, in short, "$x \in DW_{1/W^2}$". Here is the new scheme:

(3) $x \in DW_{1/W^2} \equiv$ a. $W_1 \cdot W_2 = 0$ (the types exclude one another)

b. $W_1 \subset W \cdot W_2 \subset W$ (the types are included in the set divided)

c. $\prod u \, (u \in W \rightarrow u \in F_1 \vee u \in F_2 \vee \ldots \vee$

$\vee \, u \in F_f \vee u \in Y_1 \vee u \in Y_2 \vee \ldots \vee u \in Y_g)$

(Every member of the divided class has at least one typical feature: the F-features being the properties of the type W_1; the Y-features being the properties of type W_2)

d. $x \in W$ (only members of W are considered)

e. $\prod u \, (u \in W_1 \equiv u \in F_1, F_2, \ldots, F_f)$

$\prod u \, (u \in W_2 \equiv u \in Y_1, Y_2, \ldots, Y_g)$

(F-features are typical of W_1; Y-features are typical of W_2)

f. $\sum i, j, k, p, r, s \Big[(x \in F_{i_1}, F_{j_2}, \ldots, F_{k_m}, Y_{p_1}, Y_{r_2}, \ldots, Y_{s_t}) \cdot$

$\cdot (i, j, k, m \leqslant f) \cdot$

$(p, r, s, t \leqslant g) \cdot (m \geqslant (c/b) \times t) \cdot ((c/b) \times t \in Nat \; N) \Big]$

(The members of $DW_{1/W}$ may have features of both the types but they have at least the same number of features typical of W_1 than a determined fraction c/b of the number of properties of the type W_2)

(3) is not the last of possible schemes. We shall consider one more. It is obvious that divisions of any number of subsets may be carried out. The definitional scheme to determine each of these subsets will be much more complicated than the schemes already presented. The definiendum will assume the following form: "$x \in DW_{a/W^p}$" i.e. "x is a member of the development of the type W_a being a subset of a p-fold division of W". The types $W_1, \ldots, W_p$ will exclude each other. To express this idea we must introduce the variables "W_i", "W_j" etc. and admit that for all natural numbers i, j from 1 to p: W_i excludes W_j in a similar manner we may formulate the condition b.: for all natural i from 1 to p: W_i is included in W. The conditions c. and e. entail the introduction of more refined symbolism. In (3) two kinds of properties were mentioned, namely the features of W_1 (the F-features) and the properties of W_2 (the Y-features). Now because of the multitude of types (p types in number) there must be p kinds of typical features. As the number p may be very high, it will not do to use different letters to indicate different kinds of typical features. Thence to indicate the kind of feature (the type the feature belongs to) we shall introduce special indexes. Thus all the features of the type W_1 will have index F^1, all the features of the type W_2 the index F^2, all the features of the type W_p the index F^p. All the previous indexes written lower remain unchanged. To speak of undetermined kinds of features the variables F^i, F^j etc. will be convenient, the lower indexes also being unchanged. To understand the following definitional scheme fully one has to keep in mind that different

types may have different numbers of typical features. Therefore we cannot say that for every natural i from 1 to p: W_i has m typical features, for then every W_i must have exactly the same number m of typical features. We may be able to say that each type W_i may have a different number of typical properties if we relativize the number of features to its type in the following manner: the type W_i has m^i typical features. In the case of different types the number m^i may be different. All these explanations allow to introduce the scheme:

(4) $x \in DW_{a/W^p} \equiv$

a. $\prod i, j\, [i \neq j \cdot (1 \leqslant i \leqslant p) \cdot (1 \leqslant j \leqslant p \rightarrow W_i \cdot W_j = 0)]$

(All types exclude each other)

b. $\prod i\, (1 \leqslant i \leqslant p \rightarrow W_i \subset W)$ (Every type is included in the set divided)

c. $\prod i, j, u\, [u \in W \cdot (1 \leqslant i \leqslant p) \cdot (1 \leqslant j \leqslant p) \rightarrow$

$\rightarrow u \in F_1^i \vee u \in F_2^i \vee \ldots \vee u \in F_{n^i}^i \vee$

$\vee\ u \in F_1^j \vee u \in F_2^j \vee \ldots \vee u \in F_{n^j}^j \vee \ldots \vee$

$\vee\ u \in F_1^p \vee u \in F_2^p \vee \ldots \vee u \in F_{n^p}^p)]$

(Members of W have at least some properties of some types)

d. $x \in W$

e. $\prod u, i\, [1 \leqslant i \leqslant p \rightarrow (u \in W_i \equiv u \in F_1^i,\ F_2^i, \ldots, F_{n^i}^i)]$

(Every type has n^i properties; n^i being allowed to vary with the type)

f. $\sum h, f, k, m\, \{(x \in F_{h^1}^a,\ F_{f^2}^a, \ldots, F^a{}_{k^m}) \cdot$

$(h, f, k, m \leqslant n^a) \cdot\ \sum i, r, s, t, q\, [i \neq a] \cdot$

$(1 \leqslant i \leqslant p) \cdot (t^i, q^i, r^i, s^i \leqslant n^i) \cdot$

$(x \in F^i{}_{t^{i1}}, F^i{}_{q^{i2}}, \ldots, F^i{}_{r^{is^i}}) \cdot (m \geqslant s^i)]\}$

(The member of DW_a has at least the same number of features of the type W_a than of other types)

The scheme (4) is not the last possible. It may be changed in many respects to make it apt to be applied to every situation we need. For instance we may introduce a hierarchy of features counting one feature as several other features, thus treating the first feature as more important. We may also cease to speak of having features and consider only possibilities of having features. Perhaps it may be desirable to make definitional schemes conditioned on the levels of knowledge. This dependence will clearly display the changing of meanings with the growing of our knowledge.

5. As far as I know the proposed definitional schemes have not been singled out up now though in every day practice as well as in scientific enterprises they were often used[2].

These definitional schemes seem not to have been recognized because of a strong tradition to the effect that classes may be delimited only with the help of properties which are common to all members of the defined class and only to them. That is why biologists tried again and again to define animals by finding features common to all animals and only to animals. These attempts failed. On the contrary, the proposed definitional schemes suggest that one may delimit a class without finding a single feature common to all animals and only to animals. Notice that according to (2) if we take r/p as $1/2$ and assume that there are only six typical animals features it will be enough for a living being x to have but three typical animal features to be ranked among animals. There may be another living being y having the other three typical features to be rated among the animals. So the set of animal may contain entities having no features characteristic of all animals and only animals.

We took the definition of the animal as an example to show how traditional definitional schemes run into trouble. Now let us take the same example to show these difficulties are overcome by the new schemes:

Let us assume that the typical features of animals are only the following:

F_1 motility
F_2 sensitivness
F_3 heterotrophicity
F_4 having cells with thin, elastic membranes
F_5 disability of producing chlorophil
F_6 possessing at least rudiments of alimentary canal

In view of these typical features the definition of animal is to be the following:

(5) x is an animal (is a member of the development of the animal type) $\equiv$

a. $\prod u$ (u is a member of the animal type $\rightarrow$ u is $F_1, F_2, F_3, F_4, F_5, F_6$).

b. $\sum h, i, j, k\ [(h, i, j, k \leqslant 6) \cdot (h \geqslant 3) \cdot (x \text{ is } F_{i_1}, F_{j_2}, \ldots, F_{k_h})]$

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[2] The ideas of Beckner (*Biological Way of Thought*. 1959, p. 22–25) only partly resemble our conception. Beckner states that (a) the number of typical features cannot be specified; (b) each typical feature is possessed by large number of individuals of the defined development, nothing of that kind was stated in (1)–(4) as making a definition of a development usually we do not know how large a part of it is the type; (c) no typical feature is possessed by every entity of the class defined. Here the traditionally known definitions are allowed to be treated as the limiting instances of our schemes. Beckner excludes this possibility in (c). Accepting (c) Beckner makes it impossible to expand definitions of meaning families (of polytypic concepts, in his terminology) into a Boolean sum of Boolean products (cf. p. 24). Beckner does not consider what features to take into account to define a meaning family (polytypic concept). Here the characteristic features of some typical entities (hence typical features) were proposed as those which could serve to define a polytypic concept. The last difference between (1)–(4) and Beckner's conception consists in the fact that his ideas correspond at most to (1), (2), (3) and (4) go beoyng Beckner's conception.

According to this definition the sponges and corals are animals though they created difficulties in the case of traditional definitions. Let us take into consideration a sponge displaying the properties F_2, F_3, F_4, F_5. This sponge has more than half the typical animals features. It implies that there do exist typical features $F_i, F_j, \ldots, F_m$ $(i, j, k, m \leqslant \leqslant 6)$ displayed by the sponge. What is more, the sponge has exactly four such properties and this implies in turn that there is an h such that $h \geqslant 3$ and h is the number of properties displayed by the sponge. Hence we are entitled to accept:

$$(6) \quad \sum h, i, j, k \,[(\text{the sponge has } F_{i_1}, F_{j_2}, \ldots, F_{k_h}) \cdot (h, i, j, k \leqslant 6) \cdot (h \geqslant 3)]$$

So the condition b. of (5) is fulfilled. Since the condition a. is a determination of the type which is given independently the definition (5) is fulfilled by the given sponge. It is therefore and animal q.e.d.

The scheme (1) makes it even possible to decide whether the Green Euglena ranked both to zoology and to biology is indeed an animal or not an animal. Euglena is motile (flagellatum), sensitive (stigma), its cell has a thin elastic membrane, possesses rudiments of an alimentary canal (peristoma). So the green Euglena has more than three typical animal features and must be rated among the animals. On the contrary, the mushrooms though heterotrophic are plants as they have on lytwo typical animal features: are heterotrophic and do not produce hemoglobin.

Perhaps our ranking of the green Euglena is not the best but the definitional scheme is not to be blamed for it, as we took only six properties into account. The more are the typical and mutually independent features considered, the more natural is the classification.

6. If the meaning of "animal" is really a meaning family we succeded in finding a definitional scheme which makes it possible to define terms having meaning families as their meaning. What is more, it seems that the proposed schemes faithfully record the procedures of defining many terms of science and of everyday life. Thus one of the problems put by Wittgenstein has found its positive solution. If, on the other hand, "animal" is not such a term as "game", we succeded at least in showing that there may be correct definitions where, till now, one could not find any adequate ones.

We have given several definitional schemes in a language well known and this helped to demonstrate that using logical tools one may describe procedures and meaning which sometimes are considered as describable in natural languages only. The schemes were presented also to show how many so far not mentioned possibilities may be realized if one wants to adapt the schemes to make them serve in the various situations we meet with.

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